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# RESEARCH MEMORANDUM

COOLING OF GAS TURBINES

I - EFFECTS OF ADDITION OF FINS TO BLADE TIPS  
AND ROTOR, ADMISSION OF COOLING AIR THROUGH PART OF  
NOZZLES, AND CHANGE IN THERMAL CONDUCTIVITY  
OF TURBINE COMPONENTS

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APR 16 1947  
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WASHINGTON

February 11, 1947

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NACA RM No. E7B1la

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RESEARCH MEMORANDUM

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I. - EFFECTS OF ADDITION OF FINS TO BLADE TIPS

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SUMMARY

An analysis was developed for calculating the radial temperature distribution in a gas turbine with only the temperatures of the gas and the cooling air and the surface heat-transfer coefficients known. This analysis was applied to determine the temperatures of a complete wheel of a conventional single-stage impulse exhaust-gas turbine. The temperatures were first calculated for the case of the turbine operating at design conditions of speed, gas flow, etc. and with only the customary cooling arising from exposure of the outer blade flange and one face of the rotor to the air. Calculations were next made for the case of fins applied to the outer blade flange and the rotor. Finally the effects of using part of the nozzles (from 0 to 40 percent) for supplying cooling air and the effects of varying the metal thermal conductivity from 12 to 260 Btu per hour per foot per °F on the wheel temperatures were determined. The gas temperatures at the nozzle box used in the calculations ranged from 1600° F to 2000° F.

The results showed that if more than a few hundred degrees of cooling of turbine blades are required other means than indirect cooling with fins on the rotor and outer blade flange would be necessary. The amount of cooling indicated for the type of finning used could produce some improvement in efficiency and a large increase in durability of the wheel. The results also showed that if a large difference is to exist between the effective temperature of the exhaust gas and that of the blade material, as must be the case with present turbine materials and the high

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exhaust-gas temperatures desired (2000° F and above), two alternatives are suggested: (a) If metal with a thermal conductivity comparable with copper is used, then the blade temperatures can be reduced by strong cooling at both the blade tip and root. The center of the blade will be less than 200° F hotter than the ends; (b) With low conductivity materials some method of direct cooling other than partial admission of cooling air is essential. From this study, it can be deduced that indirect cooling of turbine blades will not make possible large increases in effective gas temperature.

## INTRODUCTION

The power and the efficiency of compressor-turbine units are limited by the allowable gas temperature, which in turn is restricted by the strength and the durability of the first-stage rotor blades where the combination of high temperatures and high stresses produces the severest strains. With present blade materials, some method of blade cooling is required to permit further increases in gas temperatures.

Blades may be cooled (a) indirectly, by reducing the temperature at the blade root (wheel rim) or at the blade tip (either method causes the blade itself to be cooled by conduction); or (b) directly, by forcing air or liquid past the blade surfaces or through passages within the blade (reference 1).

In the analysis of the cooling of gas turbines, the effects of various cooling systems, types of blade, thermal conductivities, heat-transfer coefficients, and coolants on the over-all performance of compressor-turbine units and the increase in allowable gas temperature will be considered.

In the present report, an analysis for calculating the approximate radial temperature distribution in the components of a gas-turbine wheel is presented in which only the cooling-air and gas temperatures and the surface heat-transfer coefficients have to be known for application. The analysis is applied to determine the wheel temperatures for the cases of customary cooling of the outer blade flange and one face of the rotor by air and for addition of circumferential fins to the outer blade flange and the rotor. The effects of using part of the nozzles for admitting cooling air (usually called partial admission) and variation of thermal conductivity of the metal used in the wheel on the temperatures are also determined by use of the analysis. The calculations were made for

gas temperatures at the nozzle box of 1600° F, 1800° F, and 2000° F, thermal conductivities from 12 to 260 Btu per hour per foot per °F, and partial admission of cooling air using 0 percent, 13 percent, 26 percent, and 40 percent of the nozzles. Both fins and partial admission of cooling air were used simultaneously. The conductivity effect was determined first by changing only the blade metal and second by changing the metal of the whole wheel.

## SYMBOLS

|                              |  |
|------------------------------|--|
| $A_b$                        | average cross-sectional area of blade, (sq ft)   |
| $A_r$                        | average cross-sectional area of rotor, (sq ft)   |
| $B, C, D,$<br>$E, G, \Delta$ | integration constants  |
| $F$                          | effective mean temperature of cooling air and exhaust gas, (°F)  |
| $J, iH_0$                    | Bessel functions   |
| $k$                          | thermal conductivity of metal, (Btu)/(hr)(sq ft)(°F/ft)  |
| $l$                          | average blade length (excluding flange), (ft)  |
| $n$                          | ratio of number of nozzles used for cooling air to number used for exhaust gas   |
| $N$                          | number of blades   |
| $p$                          | blade perimeter, (ft)  |
| $q_i$                        | heat-transfer coefficient from exhaust gas to blade, (Btu)/(hr)(sq ft)(°F)   |
| $q_o$                        | heat-transfer coefficient from blade flange to cooling air, (Btu)/(hr)(sq ft)(°F)  |
| $q_i'$                       | apparent heat-transfer coefficient from hot air to blade flange, $\frac{2\lambda - t_b}{2\lambda} q_i$ , (Btu)/(hr)(sq ft)(°F) |

|             |   |
|-------------|---|
| $q_o$       | heat-transfer coefficient from rotor surface to cooling air, $(\text{Btu}/(\text{hr})(\text{sq ft})(^{\circ}\text{F}))$               |
| $\bar{q}_o$ | modified $q_o$ for partial admission of cooling air, $(\text{Btu}/(\text{hr})(\text{sq ft})(^{\circ}\text{F}))$                       |
| $r$         | radial distance from rotation axis, (ft)  |
| $\tau_i$    | time heat comes in from exhaust gases (partial admission), (hr)   |
| $\tau_o$    | time heat goes out to cool air, (hr)  |
| $t_b$       | average blade thickness, $(2A_b/p)$ , (ft)  |
| $t_r$       | average rotor-rim thickness, (ft)   |
| $T$         | temperature, $(^{\circ}\text{F})$   |
| $T_a$       | temperature of cooling air, $(^{\circ}\text{F})$  |
| $T_g$       | effective temperature of exhaust gases (temperature that determines heat flow from gas to blade) (ref. 2, p. 3), $(^{\circ}\text{F})$ |
| $T_r$       | temperature at wheel rim, $(^{\circ}\text{F})$  |
| $x$         | distance from blade tip to general blade point, (ft)  |
| $y$         | distance from flange tip to general flange point, (ft)  |
| $\theta$    | temperature difference between cooling air or exhaust gas and metal, $(^{\circ}\text{F})$   |
| $2\lambda$  | flange length, (ft)   |

## ANALYSIS OF TEMPERATURE DISTRIBUTION

## Blade Temperatures

The assumed blade shape is shown in figure 1(a). The area and the perimeter are assumed uniform and the thickness is assumed small in comparison with the other dimensions. At each blade tip is a flange, the outside of which is exposed to cooling air. The flange extends to the next blade to form a nearly continuous rim around the outer end of the blades. The thickness of the flange is the same as that of the blade. The assumptions used in the calculations for this blade are as follows:

- (1) The flange temperatures are approximated by the arrangement shown in figure 1.
- (2) The temperature changes in other than the radial direction are neglected.
- (3) The variations in area, perimeter, thermal conductivity, and heat-transfer coefficient are negligible for any given section; that is, constant mean values can be used.
- (4) The effect of radiation is negligible.
- (5) At the junctions of the various sections, temperature and heat flow are continuous but temperature gradients are not continuous at area discontinuities.

The flange can be considered as a plate of constant area and equations based on that assumption will give the same temperature distribution in the blade proper as will the use of assumption (1).

From considerations of symmetry, it is clear that midway between the two blades, that is, at each end of the flange,  $dT/dy = 0$  because there can be no heat flow across these sections.

An almost equivalent arrangement, as far as temperature distribution is concerned, is shown in figure 1(b), where the flange ends are folded back along the x-axis instead of along the y-axis. Thus, the heating and the cooling surfaces are the same as for the arrangement shown in figure 1(a) but the equations for the flange and the blade proper can be fitted together more easily.

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For the two elements of flange  $dx$  (fig. 1(b)) (blade width is assumed large compared with  $t_b$  so that it has the value  $p/2$ ),

$$\text{Heat entering left sides by conduction} = -k 2t_b \frac{dT}{dx} \frac{p}{2}$$

$$\text{Heat entering from hot gas} = q_i' 2dx (T_g - T) \frac{p}{2}$$

where  $q_i'$  is  $q_i$  modified for blade thickness.

$$\text{Heat leaving right side by conduction} = \left( -k 2t_b \frac{dT}{dx} - k 2t_b \frac{d^2T}{dx^2} dx \right) \frac{p}{2}$$

$$\text{Heat leaving to cooling air} = q_o 2dx (T - T_a) \frac{p}{2}$$

When the heat entering the element is equated to the heat leaving the element,

$$-k 2t_b \frac{dT}{dx} + q_i' 2dx (T_g - T) = -k 2t_b \frac{dT}{dx} - k 2t_b \frac{d^2T}{dx^2} dx + q_o 2dx (T - T_a)$$

$$\frac{d^2T}{dx^2} - \frac{q_o + q_i'}{kt_b} T = -\frac{q_o T_a + q_i' T_g}{kt_b}$$

$$\frac{d^2T}{dx^2} - \mu^2 T = -\gamma^2$$

where

$$\mu = \sqrt{\frac{q_o + q_i'}{kt_b}}$$

$$\gamma = \sqrt{\frac{q_o T_a + q_i' T_g}{kt_b}}$$

A convenient solution is

$$T = F + G \cosh \mu (x + \lambda) \quad (1)$$

where

$$F = \frac{q_o T_a + q_i' T_g}{q_o + q_i'}$$

and the assumed boundary condition on the left end is

$$dT/dx = 0$$

when

$$x = -\lambda$$

For the blade proper, from  $x = 0$  to  $x = l$ , the heat equation is

$$t_b k \frac{d^2\theta}{dx^2} dx = q_i 2dx \theta$$

where

$$\theta = T_g - T$$

or

$$\frac{d^2\theta}{dx^2} - \nu^2 \theta = 0$$

where

$$\nu^2 = \frac{2q_i}{kt_b}$$

A suitable solution of the foregoing equation is

$$T_g - T = C [\cosh \nu (x - B)] \quad (2)$$

#### Rim and Rotor Temperatures

The rim and the rotor of the turbine wheel are solid disks. The difference in shape (fig. 1(c)) lies in the fact that the rim has a uniform thickness in the axial direction, whereas the rotor is usually of constant strength with variable thickness. In many cases this variation is of such a nature that the area normal to a radius from the rim to a point where the constant-strength contour is abandoned is approximately constant; that is, as the radius increases, the thickness decreases in approximately the same proportion.

An element of the rim to be considered is  $2\pi r t_r dr$ .

$$\text{Heat entering the outer surface of element} = k 2\pi t_r (r+dr) \left[ \frac{dT}{dr} + \frac{d}{dr} \left( \frac{dT}{dr} \right) dr \right]$$

$$\text{Heat leaving from one side surface element} = (2\pi r dr) q_o' (T - T_a)$$

$$\text{Heat leaving inner surface of element} = k 2\pi t_r r \frac{dT}{dr}$$

If  $\theta$  is substituted for  $T - T_a$  and the heat leaving the element is equated to that entering, the resulting equation becomes

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} = \frac{q_o'}{kt_r} \theta$$

The solution of this equation is given in reference 3 (equations (65) and (53)) as

$$T - T_a = D J_o \left( ir \sqrt{\frac{q_o'}{kt_r}} \right) + E i H_o \left( ir \sqrt{\frac{q_o'}{kt_r}} \right) \quad (3)$$

In the variable thickness section, the flow area  $A_r$  ( $= 2\pi r t$ ) is constant resulting in the following heat balance:

Heat entering outer surface of an element  $A_r dr$  is

$$k A_r \left( \frac{d\theta}{dr} + \frac{d^2\theta}{dr^2} dr \right)$$

$$\text{Heat leaving one side} = 2\pi r dr q_o' \theta$$

$$\text{Heat leaving inner surface} = k A_r \frac{d\theta}{dr}$$

When the heat entering the element is equated to that leaving and the equation is simplified

$$\frac{d^2\theta}{dr^2} = \alpha^2 r \theta$$

where

$$\alpha^2 = \frac{2\pi q_o}{k A_r}$$

A solution is

$$\begin{aligned} \theta = T - T_a &= r^{1/2} \Delta i^{1/3} J_{-1/3} \left( \frac{2/3 \alpha i r^{3/2}}{\sqrt{\frac{2\pi q_o}{k A_r}}} \right) \\ &= r^{1/2} \Delta i^{1/3} J_{-1/3} \left( \frac{2/3 i r^{3/2}}{\sqrt{\frac{2\pi q_o}{k A_r}}} \right) \quad (4) \end{aligned}$$

after application of the boundary condition

$$\frac{dT}{dr} = 0$$

when

$$r = 0$$

These Bessel functions are tabulated in reference 4.

#### Validity of Assumptions

Equations (1), (2), (3), and (4) give the complete temperature distribution throughout the turbine in a radial direction in terms of the exhaust-gas and the cooling-air temperatures, the surface heat-transfer coefficients, thermal conductivity, and the turbine dimensions subject to the assumptions made.

The examples for which calculations were made show small temperature changes through the thickness of the blade because the blades are thin. Between the leading and the trailing edge of a turbine blade, a large temperature difference exists, in some cases 50° or 60° F, which is probably due to hot gas temperatures at the leading edge and perhaps large values of  $q_i$  caused by direct impact. The equations given are for temperature changes down the blade center, which result in mean values.

The variations in area, perimeter, conductivity, and heat transfer are not large in most cases. Four different values are permissible, one in each region; therefore, values are averaged over only a small region and not over the entire turbine. Finer subdivisions

can be used if necessary. If a large taper exists on a blade, equation (2) can be modified to agree with the law of taper. For the rotor,  $q_o$  would be expected to vary with the velocity of the air relative to the rotor. When  $q_o$  varies with  $r$  in a regular way, equation (4) can be modified.

The effect of radiation is to modify the values of  $F$ ,  $T_g$ ,  $\mu$ , and  $\nu$  in equations (1) and (2). With assumed values for a conventional type of blade, the effect on  $F$  and  $T_g$  was negligible. The effect on  $\mu$  and  $\nu$  is larger, of the order of 10 percent in one case, but allowance can be made for this effect by increasing the value of  $q_i$  by a constant amount for each temperature; therefore the form of the equation will not be affected. An estimated amount could be added to the value of  $q_i$  to correct for the radiation.

The area changes are not actually discontinuous but the fillets, as a rule, do not have large radii. Accordingly, it is simpler to use assumption (5) because the errors caused by this procedure will rapidly disappear as the distance from the junction point increases.

#### Effect of Increased Cooling on the Temperature Distribution

In the case of air-cooled engines, the over-all heat transfer is increased by attaching fins to the outside surface of the cylinders. Thus, by the addition of fins the over-all value of  $q_o$  can be increased to several times its value without fins. The same method can be used for turbine parts where there is room for the fins either on the rotor or on the blade rim. The blade proper cannot be cooled by fins but only by partial admission of cool air or by circulating cool fluids through hollow openings inside the blades. For cooling by partial admission of air, equation (2) is modified as follows: In the steady-flow state, the heat entering an element of blade length  $dx$  in time  $\tau_i$  is  $q_i p (T_g - T) \tau_i dx$ . The heat lost in time  $\tau_o$  is  $\bar{q}_o p (T - T_a) \tau_o dx$ . The total gas flow is unchanged; therefore the gas velocity and hence  $\bar{q}_o$  will be larger than before.

The difference between these two values is the net heat gained, which is equal to the difference between the heat entering one end of element  $dx$  and that leaving the other end in time  $\tau_o + \tau_i$ ,

$$p q_i \tau_i (T_g - T) - p \bar{q}_o \tau_o (T - T_a) = -k A_b (\tau_i + \tau_o) \frac{d^2 T}{dx^2}$$

or

$$\frac{d^2T}{dx^2} - M^2 T = -\Phi M^2$$

where

$$M^2 = \frac{p}{k A_b} \frac{q_i T_i + \bar{q}_o T_o}{T_i + T_o}$$

$$= \frac{p}{k A_b} q_i \frac{1 + n (\bar{q}_o / q_i)}{1 + n}$$

and

$$\Phi = \frac{q_i T_g + n \bar{q}_o T_a}{q_i + n \bar{q}_o}$$

$$= \frac{T_g + n T_a (\bar{q}_o / q_i)}{1 + n (\bar{q}_o / q_i)}$$

A solution of this equation is

$$\Phi - T = C \cosh M (x - B) \quad (5)$$

When equation (5) is substituted for equation (2), the temperature distribution can be calculated. When  $n = 0$ , equation (5) becomes identical with equation (2).

#### APPLICATION OF ANALYSIS

The foregoing analysis is used to determine the effect on blade temperatures of varying amounts of cooling applied to the flange on the blade rim, with a fixed temperature at the blade root. The following section presents the temperature distribution through the turbine as calculated for a small amount of cooling without fins, for four times as much cooling obtained with fins (limited to four to avoid bending stresses in the fins exceeding 30,000 lb/sq in.), and for partial admission of cool air both with and without fins. The dimensions of a conventional single-stage impulse exhaust turbine are used in these calculations. The turbine is assumed to be

operating at an altitude of 25,000 feet, where  $T_a = -31^\circ F$  and the ratio of sea-level pressure to altitude pressure is 2.7. Wheel speed is assumed to be 21,300 rpm.

Effect of Flange Cooling and Thermal Conductivity  
on Blade Temperatures

The following data are assumed for cooling without fins:

$$l = 1.2 \text{ inches} = 0.1 \text{ foot}$$

$$k = 14.5 \text{ (Btu)/(sq ft)(hr)(}^\circ\text{F/ft)(Vitallium)}$$

$$t_b = 0.0657 \text{ inch} = 0.00548 \text{ foot}$$

$$q_i = 44.2 \text{ (Btu)/(hr)(sq ft)(}^\circ\text{F)} \text{ (This value was obtained from reference 5 using a gas flow of 2.12 lb/sec.)}$$

$$q_i' = \frac{2\lambda - t}{2\lambda} q_i = 33.9 \text{ (Btu)/(hr)(sq ft)(}^\circ\text{F)}$$

$$T_r = 900^\circ F$$

$$T_g = 1623^\circ F \text{ (found from the nozzle-box temperature of } 2000^\circ F \text{ by using the expansion ratio 2.7 and the gas velocity relative to the rotating blades)}$$

$$q_o = 44.2 \text{ (Btu)/(hr)(sq ft)(}^\circ\text{F)} \text{ (This value was obtained from Reynolds analogy, using appropriate flat plate values of the friction coefficient.)}$$

$$\lambda = 0.135 \text{ inch} = 0.1125 \text{ foot}$$

$$v = \sqrt{\frac{(2)(44.2)}{(14.5)(0.00548)}} = 33.4 \quad (vl = 3.34)$$

$$\mu = \sqrt{\frac{44.2 + 33.9}{(14.5)(0.00548)}} = 31.08 \quad (\mu\lambda = 0.3493)$$

$$F = \frac{(-44.2)(31.0) + (33.9)(1623)}{44.2 + 33.9} = 687^\circ F$$

Equations (1) and (2) can be solved for the three constants  $C$ ,  $B$ , and  $G$  from the boundary conditions when  $x = l$ ,  $T = T_r$ , and when  $x = 0$ ,  $T$  (from equation (1)) =  $T$  (from equation (2)) and the heat flowing out of the flange is equal to the heat flowing into the blade, that is,

$$2 t_b \left( \frac{dT}{dx} \right)_1 = t_b \left( \frac{dT}{dx} \right)_2$$

or

$$2 G \mu \sinh \mu \lambda = C v \sinh v B \quad (6)$$

The first two conditions give

$$T_g - T_r = C [ \cosh v (l - B) ] \quad (7)$$

and

$$F + G \cosh \mu \lambda = T_g - C \cosh B \quad (8)$$

The elimination of  $C$  and  $G$  by the use of equations (6) and (7) gives

$$F + \left[ \frac{T_g - T_r}{\cosh v (l - B)} \right] \left[ \frac{v \sinh v B}{2 \mu \tanh \mu \lambda} \right] = T_g - \frac{(T_g - T_r) \cosh v B}{\cosh v (l - B)} \quad (9)$$

which is solved for  $B$

$$\tanh v B = \frac{\frac{T_g - F}{T_g - T_r} \cosh v l - 1}{\frac{v}{2 \mu \tanh \mu \lambda} + \frac{T_g - F}{T_g - T_r} \sinh v l}$$

When the assumed values are substituted

$$B = 0.402 l \text{ or } 0.0402 \text{ foot}$$

$$C = 192.6^\circ F$$

$$G = 516^\circ F$$

When these values are substituted in equations (1) and (2), the curve for the blade temperatures obtained without fins can be found. It is plotted in figure 2 as a solid curve. The other three solid curves were found by setting

$$q_o = 0 \text{ (without flange cooling)}$$

$$q_o = 4 \times 44.2 \text{ (fins on flange and free-air cooling)}$$

$$q_o = \infty \text{ (infinite cooling; limiting case for flange cooling)}$$

Other amounts of cooling would give rise to intermediate curves.

A similar set of curves (dashed lines in fig. 2) was computed with twice the value for the heat-transfer coefficient, that is, with  $q_i = 88.4$  and  $q_i' = 67.8$ . A temperature-distribution curve was computed for copper, a high-conductivity ( $k = 260$ ) metal, using a high heat-transfer coefficient ( $q_i = 88.4$ ). Other conditions were the same (fins on the flange). The curve is also shown in figure 2.

#### Temperature Distribution through the Turbine

The assumed conditions for the turbine wheel are as follows:

number of nozzles = 38

$$A_r = 14.96 \text{ square inches} = 0.1039 \text{ square foot}$$

$$k = 12.1 \text{ (Btu)/(hr)(sq ft)(}^{\circ}\text{F/ft}) \text{ (owing to lower temperature of rotor)}$$

$$q_o' = 36.7 \text{ (Btu)/(hr)(sq ft)(}^{\circ}\text{F}) \text{ (This value is less than } q_o \text{ because the velocity of this part of the wheel is less than that of the blade tips.)}$$

$$t_r = 0.69 \text{ inch} = 0.0575 \text{ foot}$$

$$\frac{\text{area of outer rotor section}}{\text{area of blade}} = 3.330$$

$$\frac{\text{area of equal-strength section of rotor}}{\text{area of inner rotor section}} = 0.822$$

The radial distance at the point where the constant-area section joins the constant-thickness section is 4.2 inches or 0.3500 foot. The point where the blades leave the constant-thickness rim is 0.3983 foot. The value of  $x$  is  $0.4983 - r$ . At the three junction points,  $r = 0.3500$ ,  $r = 0.3983$ ,  $r = 0.4983$  foot, the temperatures and the heat flows must be continuous; therefore, six equations are obtained to determine the six integration constants  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $G$ , and  $\Delta$ .

The results of these calculations are shown in figures 3 to 5. The curves for cooling without fins were computed using the values  $q_o = 44.2$ ,  $q_o' = 36.7$ , and  $q_i = 44.2$ . The curves for cooling with fins were computed using the values  $q_o = 4 \times 44.2$ ,  $q_o' = 4 \times 36.7$ , and  $q_i = 44.2$ . The three exhaust-gas temperatures used were  $2000^{\circ}$  F,  $1800^{\circ}$  F, and  $1600^{\circ}$  F, which correspond to effective exhaust-gas temperatures  $T_g$  of  $1623^{\circ}$  F,  $1452^{\circ}$  F, and  $1281^{\circ}$  F, respectively. By substitution of equation (5) for equation (2), the effect of partial admission of cooling air can be found. This effect is shown in figures 4 and 5.

Three additional calculations were made using a value of  $q_i = 2 \times 44.2$ ; these curves are plotted in figure 6 and show the temperature distribution through the turbine for various values of  $k$  in the rotor and the blades. The curve with the highest peak uses  $k = 12$  in the rotor and  $k = 14.5$  in the blade; the next has  $k = 12$  in the rotor and  $k = 260$  in the blade; the curve with the lowest peak uses  $k = 260$  throughout the turbine. These last calculations were made to find what could be expected from copper in an unfavorable case.

#### DISCUSSION OF CURVES

Figure 2 shows that, when no cooling is used on the flange, the blade tip is the hottest spot, and the temperature of this hot spot is quite near the effective exhaust-gas temperature. Whenever cooling is applied to the flange, the hottest spot is a point not far from the blade center, the exact location of which depends on the amount of flange cooling used. The more flange cooling that is applied, the nearer the hot spot moves toward the blade root. When infinite cooling is applied, the hot spot is about three-eighths of the blade length from the root. The high heat-transfer coefficient raises the hot-spot temperature considerably; with infinite cooling from  $1210^{\circ}$  F to  $1410^{\circ}$  F. When copper ( $k = 260$ ) is substituted for Vitallium ( $k = 14.5$ ), the temperature through the blade becomes

much more nearly uniform and the hot spot is hardly noticeable, being only slightly hotter than the outer half of the blade. The hot-spot temperature is about  $1070^{\circ}$  F as compared with  $1480^{\circ}$  F for Vitallium ( $q_i = 88.4$  and fins on flange).

The curves of temperature distribution throughout the entire turbine (figs. 3, 5, and 6) show that the fins on the wheel cool it very markedly, close to the cooling-air temperature near the center. The wheel rim (blade root) can also be kept fairly cool —  $800^{\circ}$  F with fins for the hottest exhaust-gas temperature ( $2000^{\circ}$  F) (fig. 5) as compared with  $990^{\circ}$  F without fins (fig. 4). The hot spot is cooler in the first case ( $1310^{\circ}$  F) than in the second ( $1450^{\circ}$  F). If cooling is expressed as a percentage of the temperature difference between the hot gas and the cooling air, these values become 8-percent cooling for the hot spot and 11-percent cooling for the blade root.

When cooling by partial admission using 40 percent of the nozzles and a gas temperature of  $2000^{\circ}$  F, the rim may be cooled from  $990^{\circ}$  F to  $760^{\circ}$  F and the hot spot from  $1450^{\circ}$  F to  $1080^{\circ}$  F (fig. 4). If both fins and partial admission are used, the wheel rim (blade root) is cooled from  $990^{\circ}$  F to nearly  $600^{\circ}$  F and the hot spot from  $1450^{\circ}$  F to nearly  $1000^{\circ}$  F. Thus fins on the rotor cool the blade root satisfactorily but are not so effective on the hot spot. On the other hand, partial admission of cooling air tends to cool the hot spot better than the finning but is not so effective on the blade root.

If more than a few hundred degrees Fahrenheit of cooling are required, other means than finning would probably be necessary. The amounts indicated here could produce some improvement in efficiency and a large increase in durability of the wheel.

#### SUMMARY OF RESULTS

The following results were derived from calculations based on an analysis of the cooling of an exhaust-gas turbine by addition of fins, partial admission of cooling air, and change in thermal conductivity of turbine components:

1. The cooling due to the addition of circumferential fins is:

| Exhaust-gas temperature (°F) | Cooling at the hottest point |                      | Cooling at the blade root |                      |
|------------------------------|------------------------------|----------------------|---------------------------|----------------------|
|                              | (°F)                         | percent <sup>a</sup> | (°F)                      | percent <sup>a</sup> |
| 1600                         | 100                          | 8                    | 150                       | 11                   |
| 1800                         | 120                          | 8                    | 160                       | 11                   |
| 2000                         | 140                          | 8                    | 190                       | 11                   |

temperature decrease

<sup>a</sup>Percent cooling =  $\frac{\text{effective gas temperature} - \text{air temperature}}{\text{effective gas temperature}}$

2. The cooling due to partial admission of cooling air for an exhaust-gas temperature of 2000° F is:

| Nozzles used for cooling (percent) | Cooling at the hottest point |         | Cooling at the blade root |         |
|------------------------------------|------------------------------|---------|---------------------------|---------|
|                                    | (°F)                         | percent | (°F)                      | percent |
| 13                                 | 110                          | 7       | 50                        | 3       |
| 26                                 | 230                          | 14      | 140                       | 8       |
| 40                                 | 370                          | 22      | 220                       | 13      |

3. When both cooling fins and partial admission of cooling air are used at the same time, the resultant cooling is slightly less than the sum of the separate amounts, that is, 210° F and 350° F for the first two cases compared with 250° F and 370° F.

#### CONCLUSIONS

1. If more than a few hundred degrees of cooling of turbine blades are required, other means than indirect cooling with fins on the rotor and outer blade flange would be necessary.

2. The amount of cooling indicated for the type of finning used could produce some improvement in efficiency and a large increase in durability of the wheel.

3. If a large difference is to exist between the effective temperature of the exhaust gas and that of the blade material, as must be the case with present turbine materials and the high exhaust-gas temperatures desired ( $2000^{\circ}$  F and above), two alternatives are suggested:

(a) If metal with a thermal conductivity comparable with copper is used, then the blade temperatures can be reduced by strong cooling at both the blade tip and root. The center of the blade will be less than  $200^{\circ}$  F hotter than the ends.

(b) With low conductivity materials some method of direct cooling other than partial admission of cooling air is essential.

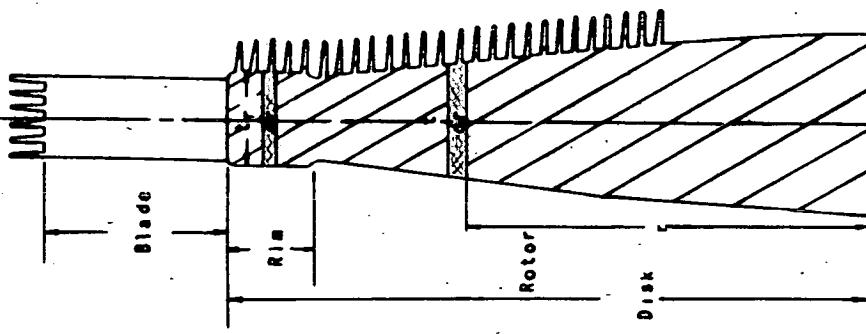
4. From the present study, it can be deduced that indirect cooling of turbine blades will not make possible large increases in effective gas temperature.

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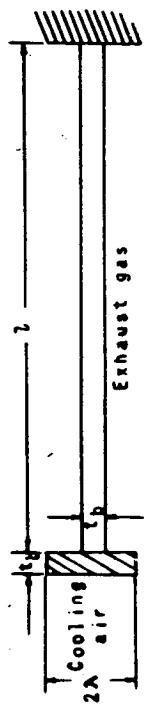
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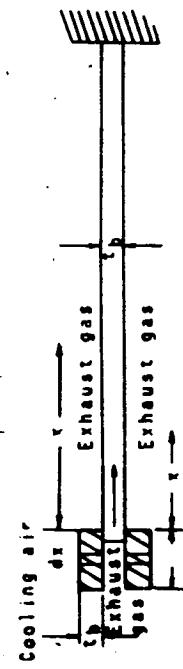
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(c) Turbine wheel showing fins.



(a) Turbine blade with flange projected along y-axis.



(b) Turbine blade with flange folded back along x-axis.

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Figure 1. Diagrammatic sketch of turbine blades and rotor.

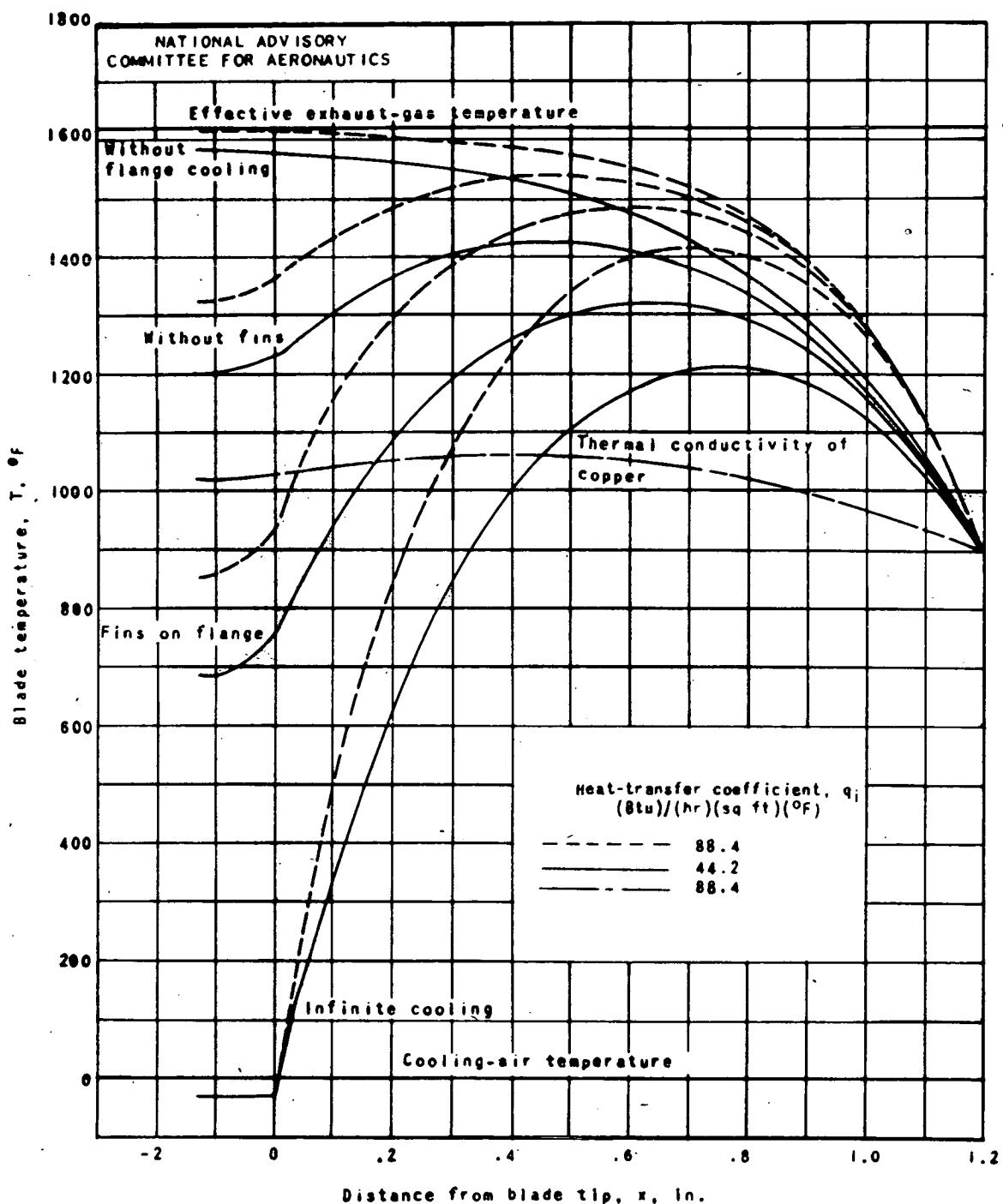


Figure 2. - Temperature distribution of turbine blade with and without fins for an exhaust-gas temperature of  $2000^{\circ}$  F.

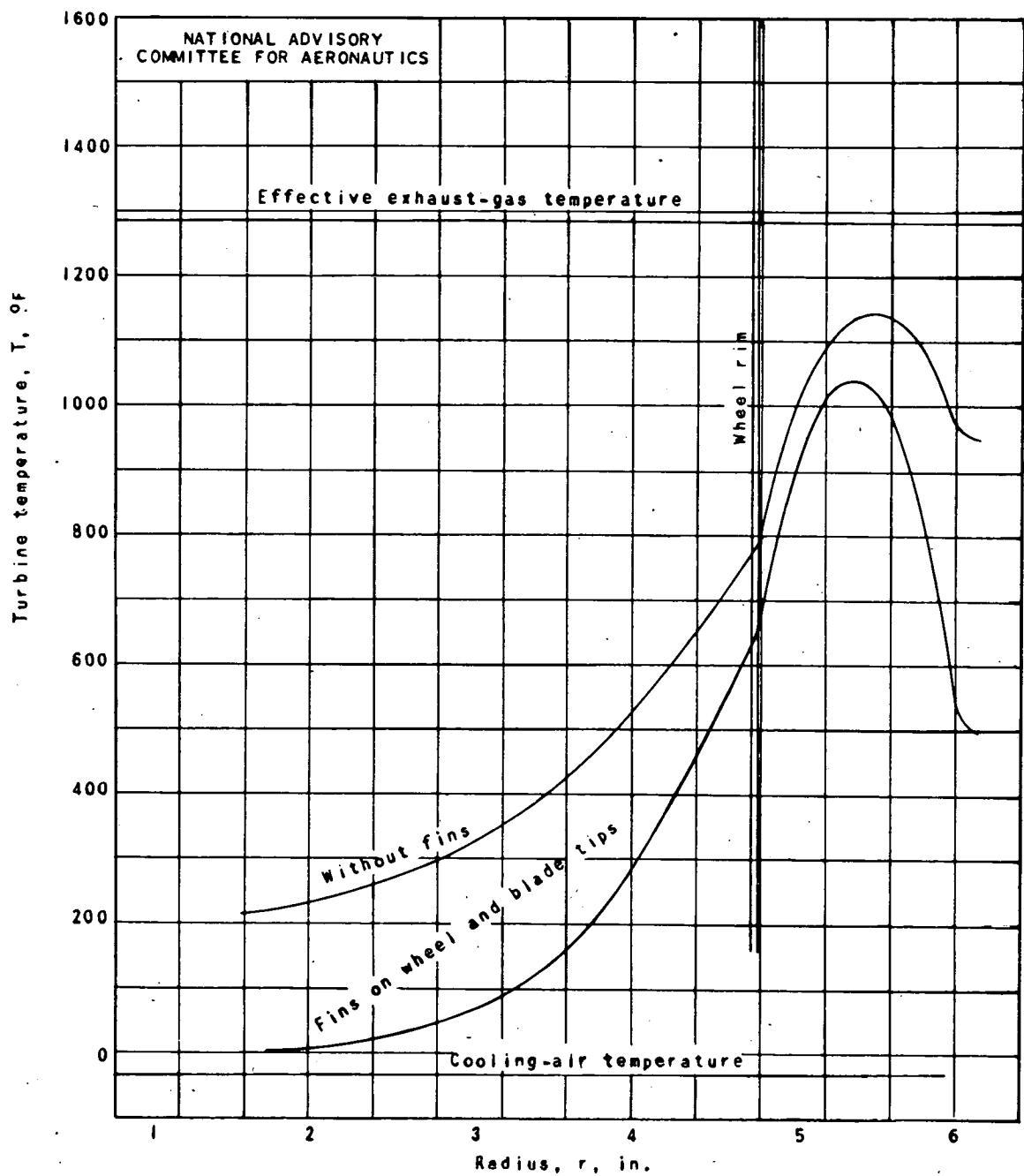
(a) Exhaust-gas temperature,  $1600^{\circ}$  F.

Figure 3. - Temperature distribution in rotor and blades.

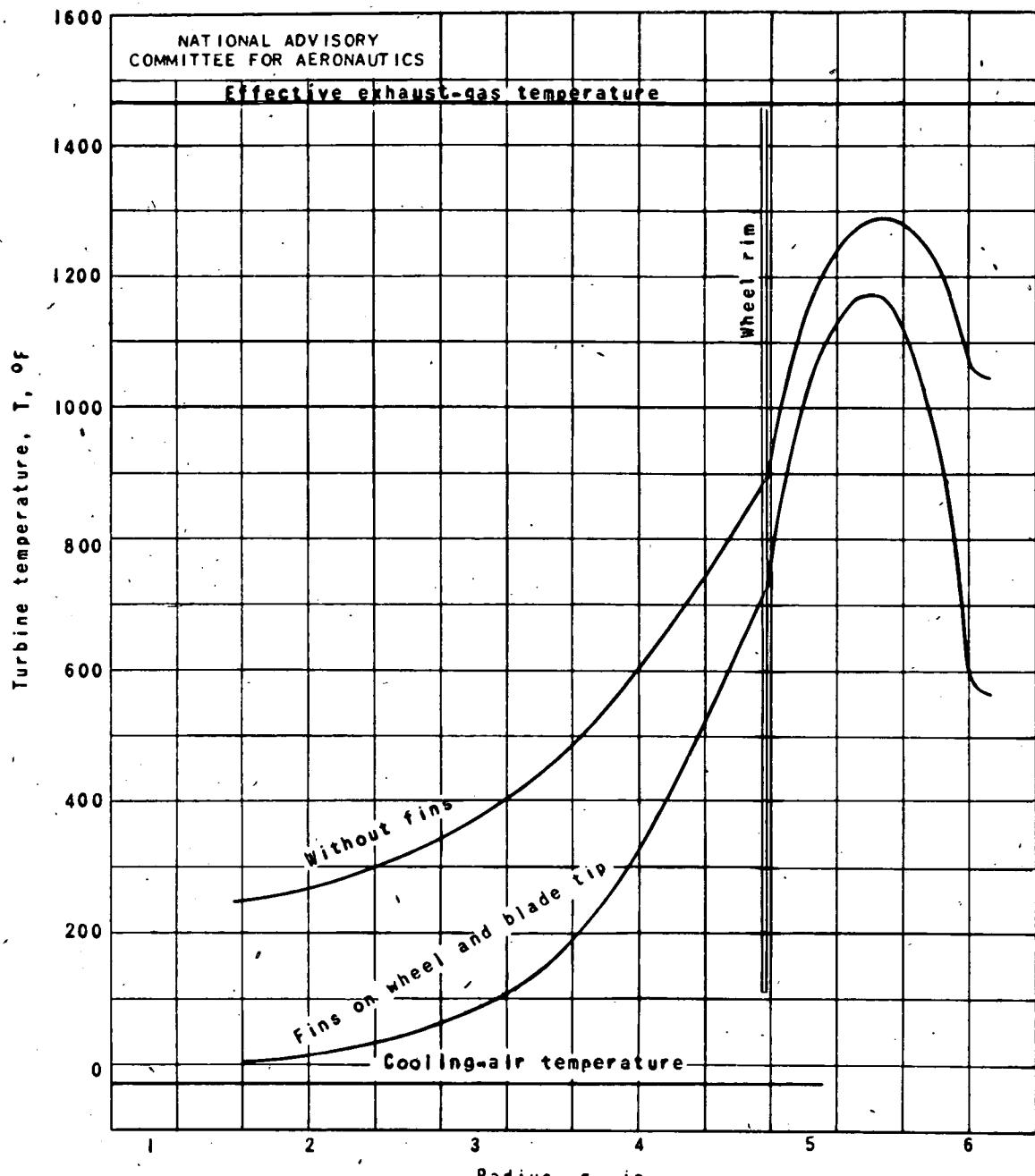
(b) Exhaust-gas temperature,  $1800^{\circ}$  F.

Figure 3. - Concluded. Temperature distribution in rotor and blades.

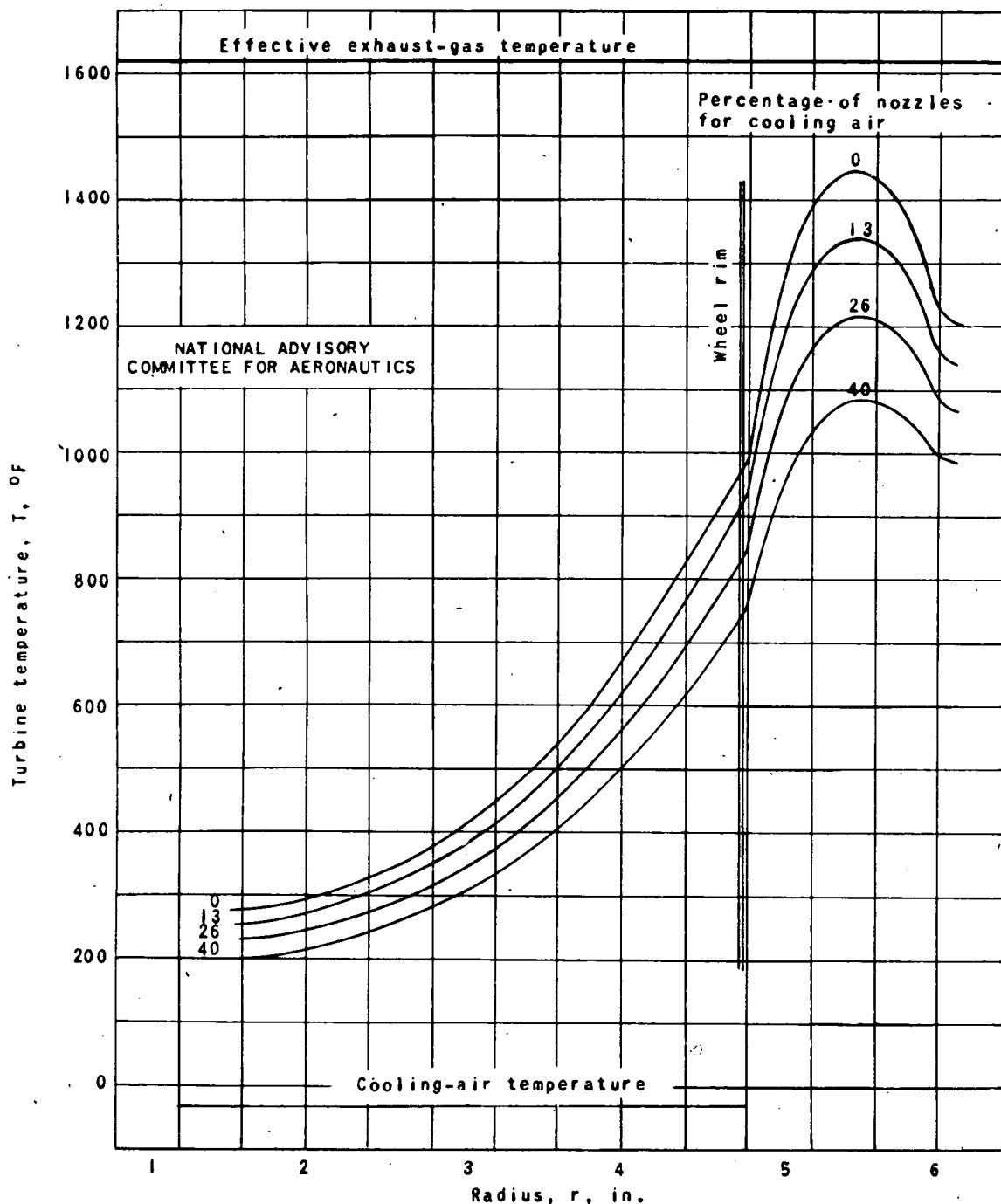


Figure 4. - Temperature distribution through turbine with partial admission of cooling air and without fins. Exhaust-gas temperature,  $2000^{\circ}$  F.

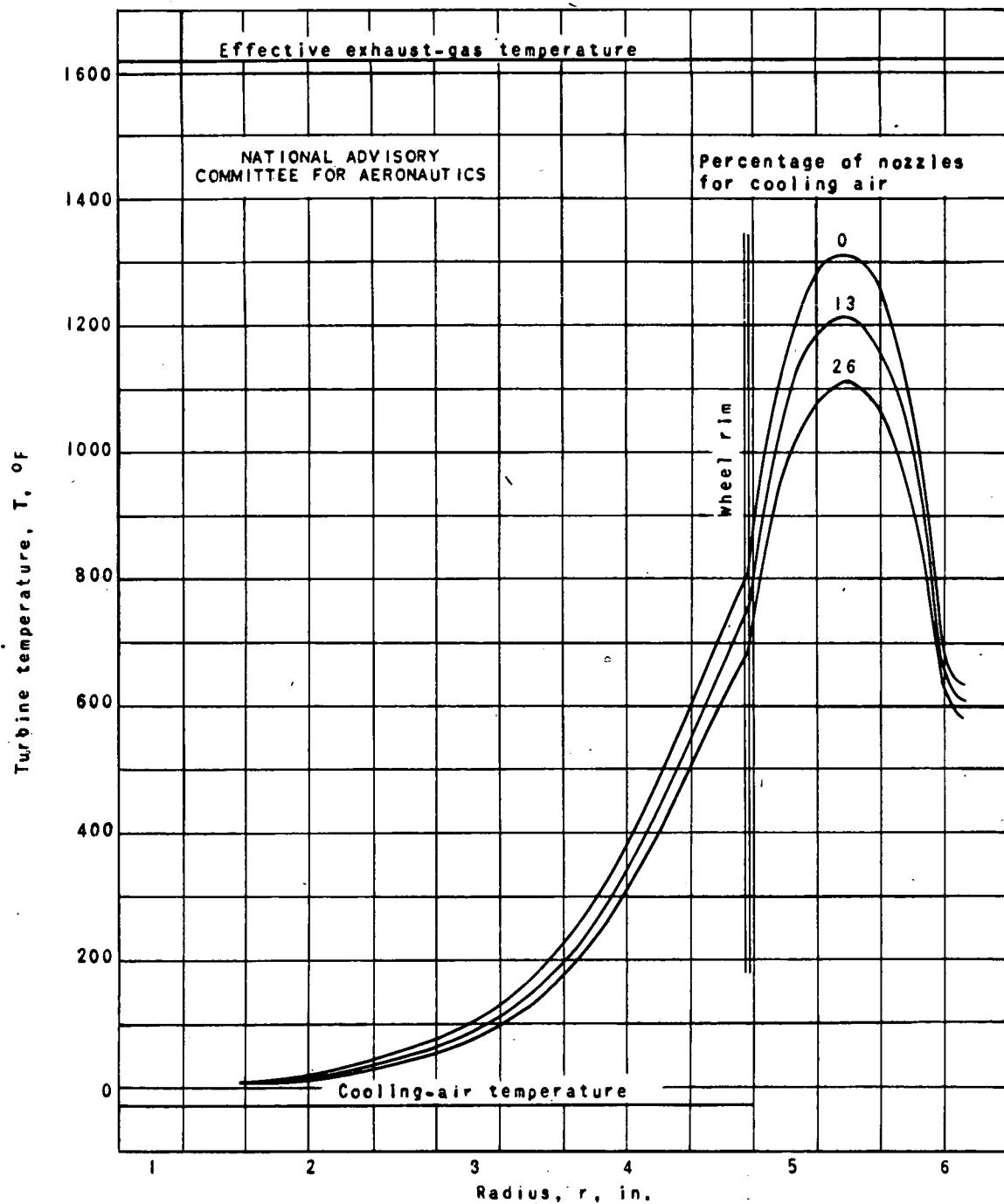


Figure 5. - Temperature distribution through turbine with partial admission of cooling air and with fins. Exhaust-gas temperature,  $2000^{\circ}$  F.

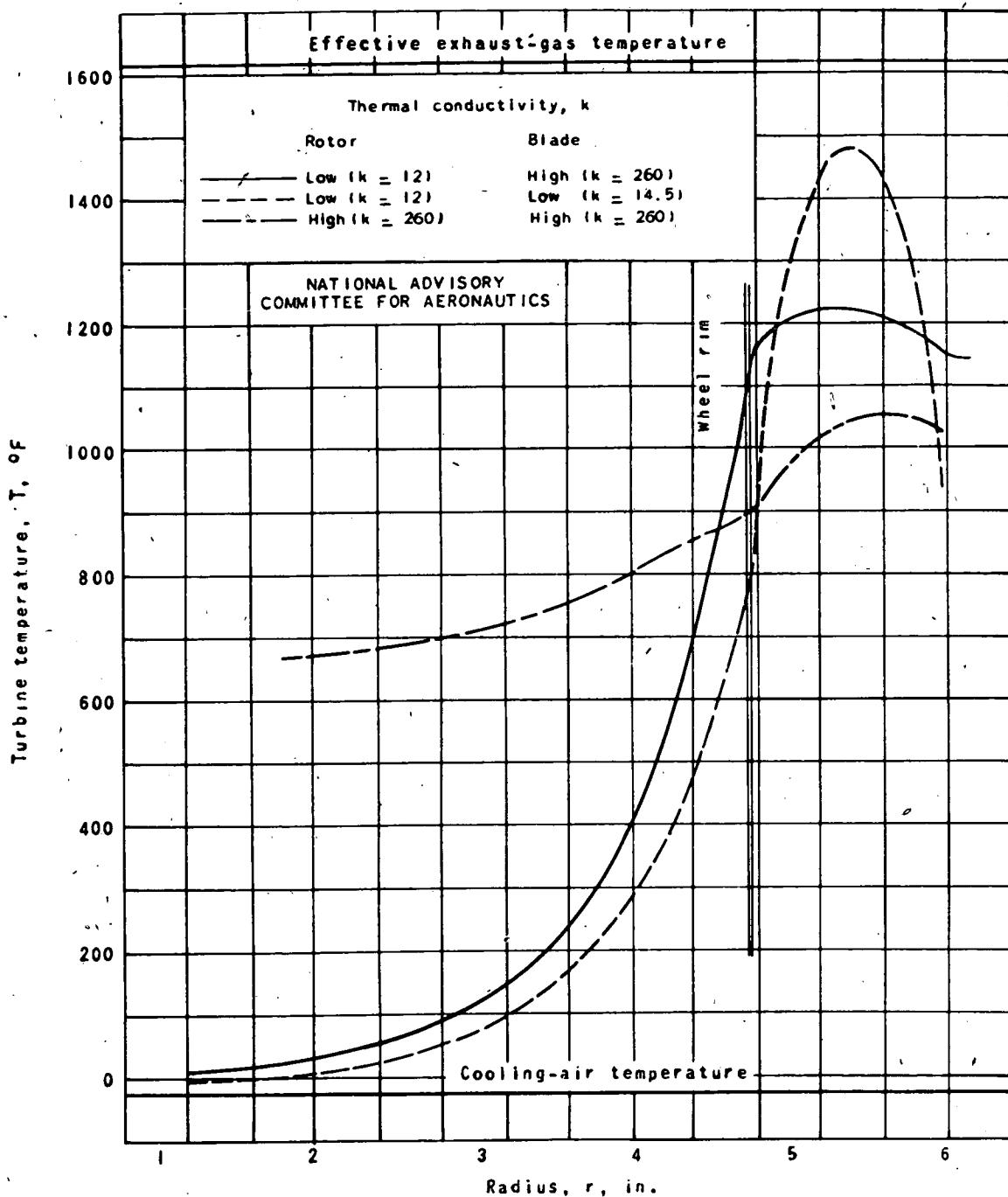


Figure 6. - Temperature distribution through turbine with high values of heat-transfer coefficient ( $q_i = 2 \times 44.2$ ), cooling fins, and rotor and blade of different thermal conductivity.

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